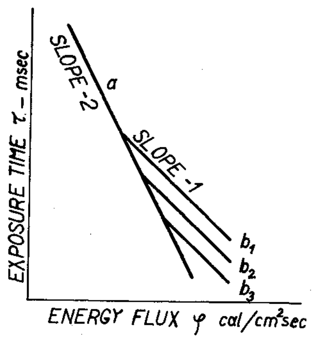

Fig. 3 Ignition characteristics of propellant.

there must be, at minimal ignition,

$$\tau = q_{in}/\varphi \tag{10}$$

In the region of minimal ignition, the slope of the line $\log\tau = f(\log\varphi)$ must, therefore, equal -1. Because at minimal ignition the temperature of the surface is higher than T_{in}, surface exotherm must appear.

Up to the present time, we have supposed that the energy flux supplied to the propellant surface is thoroughly used for heating up the solid phase of the propellant and that no liquid interlayer is produced, the formation and heating of which would require certain energy. In case of the formation of the liquid layer, a part of the energy is used for its formation and heating. This fact, however, does not change basic conclusions and results only in the fact that the slopes of the function $\log\tau = f(\log\varphi)$ will not be exactly -2 and -1, but will somewhat differ from these values. Besides that, the values of φ_K and τ_K will change as well. If the thickness of the liquid layer is dependent upon pressure, extended ignition also depends upon pressure to the degree to which heat, accumulated in the liquid layer, is a function of pressure.

It is further supposed that at minimal ignition the total heat needed for ignition q_{in} equals the enthalpy of the wave of combustion in the condensed phase at a stable flame;

$$q_{in} = (\lambda/u)(T_s - T_0) + q_e \tag{11}$$

The first member represents the enthalpy of the solid phase and the second the heat accumulated in the liquid phase of the burning propellant. Minimal ignition heat is dependent on the rate of burning in the steady state. As the velocity of burning of propellant usually depends upon pressure, the exposure time, also, will depend upon pressure as far as minimal ignition is concerned.

The dependence of exposure time as a function of heat flux consists of two different parts (Fig. 3), and in logarithmic coordinates it can be represented using a straight line a and a set of lines $b_1, b_2 \ldots b_n$, depending upon the rate of steady burning. Slope -2 is in correspondence with extended ignition and the straight line a is common to all pressures if excess enthalpy does not depend on pressure. Straights $b_1, b_2 \ldots b_n$ correspond to minimal ignition with the -1 slope. To each pressure belongs other straight $b_1, b_2 \ldots b_n$, depending upon the rate of stable burning of propellants. As with increasing pressure the rate of stable burning increases as well, the straights b are shifted, in agreement with increasing pressure, towards the left.

On the previous assumption, it is then easy to explain the dependences observed. At the same time, it is shown that the indication of ignition by means of appearing flame (or increased conductivity or pressure) is not very reliable. Flame can only serve as a proof of surface decomposition which, however, need not be stable in the case of a propellant not warmed up to a sufficient depth.

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Reply by Author to J. Pantoflíček and F. Lébr

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It is gratifying that the work reported by Pantoflíček and Lébr confirms my previous observations. Their explanation of the results involving the surface temperature gradient and the relationships between "extended ignition" (pressure-independent regime) and "minimal ignition" (pressure-dependent regime) are consistent with my views of an ignition model. Furthermore, essentially the same model has been suggested by von Elbe.[1]

However, I question the validity of the authors' suggestion that the $\log\tau$-$\log\varphi$ relationship in the region of minimal ignition is expressed by a line of slope equal to -1. Although much of my data do approximate such a relationship, some data appear to tend toward a zero slope relationship as the incident flux becomes increasingly larger than the critical flux. Perhaps such deviation from the behavior predicted by the authors can be accounted for by adequate treatment of the surface-coupled exotherm and the ablative-type endothermic reactions which might also be involved at the higher energy fluxes.

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Comment on "Forces on Spheres inside Diffusers"

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In Ref. 1, the authors presented a relationship between the force coefficient and the Reynolds number for spheres placed inside diffusers. This relationship was obtained by

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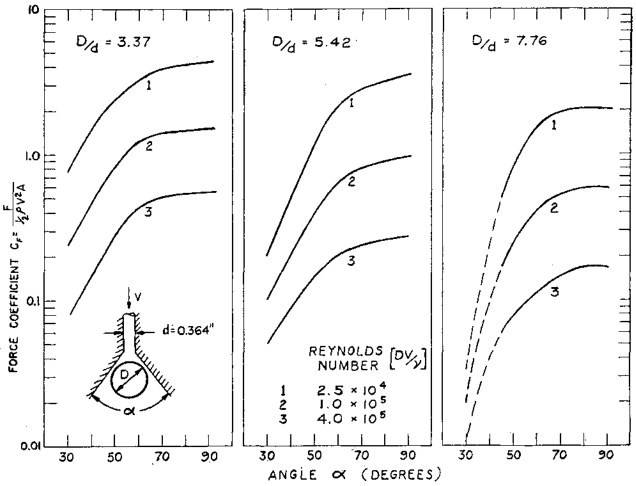

Fig. 1 Force coefficients for spheres inside diffusers.

keeping the weights of the spheres approximately constant. For this reason the relationship given in Ref. 1 for the force coefficient C_F (although correct for a given weight) implied that the force coefficient is independent of both the diffuser angle α and the diameter of the sphere D. Actually, C_F depends on both α and D, as pointed out to the authors by Dr. D. Brown, National Research Council of Canada. Data showing the effects of the diffuser angle and the sphere diameter on C_F were obtained by varying the weights of different diameter spheres. The results of these experiments are given in Fig. 1.

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Comment on "Further Study on 'A Stability Criterion for Panel Flutter via the Second Method of Liapunov'"

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Reference 1 considers the panel flutter problem for which P. C. Parks[2] developed a stability criterion using Liapunov's second method. Conditions for instability of the panel are derived. Due to an error in the calculations, however, the instability region shown in Fig. 1 of Ref. 1 cannot be deduced from the analysis of the authors, and only the region of stability is valid.

The derivation of Eq. (3) in Ref. 1 involves the inequality

$$\lambda d \int_0^1 z_{,xx}{}^2 dx \geqslant \lambda d \pi^2 \int_0^1 z_{,x}{}^2 dx$$

where

$$z = z_{,xx} = 0 \qquad \text{at} \quad x = 0 \quad x = 1$$

This inequality is only valid if $\lambda d \geqslant 0$. Since $d > 0$, it follows that (3) is not applicable if λ is chosen to be negative. The instability conditions derived in the article are based on the use of (3) with a negative value of λ, and therefore they are incorrect. No conclusions regarding instability of the panel can be obtained from the functional f used in the article.

The method considered in Refs. 1 and 2 has been used by the author[3] to obtain stability regions for flutter of the following systems: two-dimensional curved panels; rectangular simply-supported plates and cylindrical panels; and clamped plates of rectangular, circular, or almost-circular shape.

References

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Comment on "Further Study on 'A Stability Criterion for Panel Flutter via the Second Method of Liapunov'"

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Using the instability theorem of Movchan's extension of Liapunov stability criteria to continuous systems, G. R. Webb et al.[1] attempt to show that the panel is unstable for $g < -\pi^2 d$. In their investigation they use the negative of the Liapunov functional f given by Parks[2] to obtain for $\lambda < 0$

$$\frac{df}{dt} = \int_0^1 [\lambda(dz_{,xx}{}^2 + gz_{,x}{}^2) + Mz_{,x}\dot{z} + (1 - \lambda\mu)\dot{z}^2]dx$$

From which, using the Rayleigh inequality

$$d \int_0^1 z_{,xx}{}^2 dx \geqslant \pi^2 d \int_0^1 z_{,x}{}^2 dx$$

they deduce

$$\frac{df}{dt} \geqslant \int_0^1 [\lambda(\pi^2 d + g)z_{,x}{}^2 + Mz_{,x}\dot{z} + (1 - \lambda\mu)\dot{z}^2]dx$$

and $df/dt > 0$ for $g < -\pi^2 d$. In fact for $\lambda < 0$, the Rayleigh inequality becomes

$$\lambda d \int_0^1 z_{,xx}{}^2 dx \leqslant \pi^2 \lambda d \int_0^1 z_{,x}{}^2 dx$$

and so their deductions that $df/dt > 0$, and of instability for $g < -\pi^2 d$, are not proved.

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